MATH 2A/5A Prep: Exponential and Logarithm Functions

1. Determine whether $3\ln(3) - 2\ln(6)$ is positive or negative.

Solution:

$$3\ln(3) - 2\ln(6) = \ln(3^3) - \ln(6^2) = \ln(27) - \ln(36) = \ln\left(\frac{27}{36}\right)$$

Now $\frac{27}{36} < 1$, so by the graph of $y = \ln(x)$, we know $\ln\left(\frac{27}{36}\right) < 0$.

2. Order the numbers e^{4+4}, $e^{4\cdot 4}$ and e^{4^3} from smallest to largest.

Solution:

$$e^{4+4} = e^8$$
$$e^{4\cdot 4} = e^{16}$$
$$e^{4^3} = e^{64}$$

Using the graph of $y = e^x$ and the positions of 8, 16 and 64 on x axis, we know that

$$e^8 < e^{16} < e^{64}.$$

So

$$e^{4+4} < e^{4\cdot 4} < e^{4^3}.$$

3. Solve the equation $e^{2x} = 9$.

Solution: Take logarithm on both sides of the equation, we get

$$\begin{aligned} \ln(e^{2x}) &= \ln(9) \\ 2x &= \ln(3^2) \\ 2x &= 2\ln(3) \\ x &= \ln(3) \end{aligned}$$

So the solution is $x = \ln(3)$.

Note: If you somehow get $x = \ln(-3)$, this is not a solution because $\ln(-3)$ is undefined.

4. Solve the equation $\ln(1+x) - \ln(1-x) = 1$.

Solution:

$$\begin{aligned}
\ln(1+x) - \ln(1-x) &= 1 \\
\ln\left(\frac{1+x}{1-x}\right) &= 1 \\
\frac{1+x}{1-x} &= e^1 = e \text{ (by taking exponential of both sides)} \\
1 + x &= e(1-x) \\
x + ex &= e - 1 \\
(e+1)x &= e - 1 \\
x &= \frac{e-1}{e+1}
\end{aligned}$$